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(Residential Autonomous College under University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2014

FIRST YEAR

Date : 26/05/2014 **Mathematics for Economics (General)**

Time : 11 am – 2 pm Paper : II Full Marks : 75

(<u>Use separate answer books for each group</u>)

<u>Group – A</u>

1. Answer **<u>any eight</u>** questions of the following :- [8 × 5]

a) Show that $\underset{x\to 0}{\mathrm{Lt}}\,[x]$ does not exist.

b) Let $f:D\to\mathbb{R}$ be a real valued function where $D\subset\mathbb{R}$. If c be an isolated point of D then show that f is continuous at c.

c) A function $f:\mathbb{R}\to\mathbb{R}$ is defined by,
$f(x)=2x, x\in Q$ i.e. x is rational
$=1-x, x\in\mathbb{R}-Q$ i.e. x is irrational prove that f is continuous at $\frac{1}{3}$ and discontinuous at every other point in $\mathbb{R}$.

d) Let $f:\mathbb{R}\to\mathbb{R}$ be continuous on $\mathbb{R}$ and let $c\in\mathbb{R}$ such that $f(c)>\mu$. Prove that there exist a neighbourhood U of C such that $f(x)>\mu$ for all $x\in U$.

e) A function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=|x+1|+|x-1|, x\in\mathbb{R}$. Find the derived function f'.

f) State and prove Cauchy's mean value theorem.

g) Evaluate $\underset{x\to 0}{\mathrm{Lt}}\left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$.

h) If f is differentiable on [0, 1], show by cauchy mean value theorem that the equation $f(1)-f(0)=\frac{f'(x)}{2x}$ has at least one solution in (0, 1).

i) A function $f:\mathbb{R}\to\mathbb{R}$ satisfies the condition $|f(x)-f(y)|\le(x-y)^2$, for all $x,y\in\mathbb{R}$. Prove that f is a constant function on $\mathbb{R}$.

j) Use Taylor theorem to prove that, $1+\frac{x}{2}-\frac{x^3}{8}<\sqrt{1+x}<1+\frac{x}{2}$, if $x>0$.

k) Let $f:I\to\mathbb{R}$, (where I is an interval) be such a function that f has a local extremum at an interior point c of I. If $f'(c)$ exist then prove that $f'(c)=0$.

l) Find the points of local maximum and local minimum of the function $f:[0,3]\to\mathbb{R}$ defined by, $f(x)=|x-1|+|x-2|$.

<u>Group – B</u>

2. Answer **<u>any five</u>** questions of the following :- [5 × 5]

a) Reduce the matrix to the fully reduced normal form and find the rank of the matrix $\begin{pmatrix} 2 & 4 & 1 & 0 \\ 1 & 2 & 0 & 3 \\ 3 & 6 & 2 & 5 \end{pmatrix}$.

b) Find the matrix A if $A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ 1 & -2 & 3 \\ 3 & -3 & 4 \end{pmatrix}$.

c) State and prove the necessary and sufficient condition for a homogeneous system to have non zero solutions.

d) Prove that the rank of a matrix remains invarient under an elementary row operation.

e) For what values of a the system of equations is consistent? Solve completely for one such value of a .

$$\begin{aligned} x - y + z &= 1 \\ x + 2y + 4z &= a \\ x + 4y + 6z &= a^2 \end{aligned}.$$

f) If $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ find the rank of the matrix $A + A^2 + A^3$.

g) Find a particular integral of the difference equation $u_{x+2} - 8u_{x+1} + 25u_x = 2x^2 + x + 1$.

h) Find the general solution of $u_{x+2} + u_{x+1} - 12u_x = 5^x,\ x \geq 1$.

3. Answer **any two** questions of the following :- [2 × 5]

a) If $5x\dfrac{dy}{dx} = 6 - y - y^2$ and y = 1 when x = 2 find y when x = 4.

b) Reduce the given differential equation to Bernoulli's equation and solve it : $xy - \dfrac{dy}{dx} = y^3 e^{-x^2}$.

c) Sove $(xy^2 - e^{1/x^2})dx - x^2 y dy = 0$.

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